Choose the term from above to complete each sentence.

1. A feasible region that is open and can go on forever is called__________.

SOLUTION:

unbounded

2. To _________means to seek the best price or profit using linear programming.

SOLUTION:

Optimize

3. A matrix that contains the constants in a system of equations is called a(n) ______________.

SOLUTION:

constant matrix

4. A matrix can be multiplied by a constant called a(n) __________________________.

SOLUTION:

scalar

5. The _________ of a matrix with 4 rows and 3 columns are 4×3.

SOLUTION:

dimensions

6. A system of equations is _________if it has at least one solution.

SOLUTION:

consistent

7. The ______________ matrix is a square matrix that, when multiplied by another matrix, equals that same matrix.

SOLUTION:

identity

8. The _________is the point at which the income equals the cost.

SOLUTION:

break-even point

9. A system of equations is __________ if it has no solutions.

SOLUTION:

inconsistent

10. If the product of two matrices is the identity matrix, they are _______________.

SOLUTION:

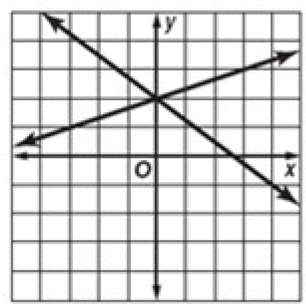
inverses

Solve each system of equations by graphing.

11. $3x + 4y = 8$
$x - 3y = -6$

SOLUTION:

Graph both equations on the coordinate plane.

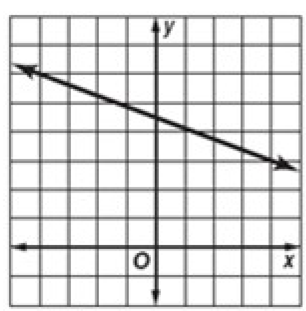


The solution of the system is (0, 2).

12. $x + \frac{8}{3}y = 12$
$\frac{1}{2}x + \frac{4}{3}y = 6$

SOLUTION:

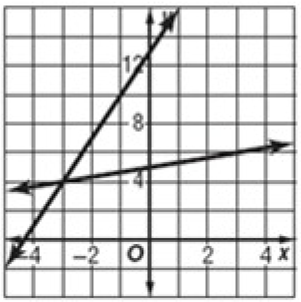
Graph both equations on the coordinate plane.



Since the graph of both the equations coincides, the system has infinitely many solutions.

13. $y - 3x = 13$
$y = \frac{1}{3}x + 5$

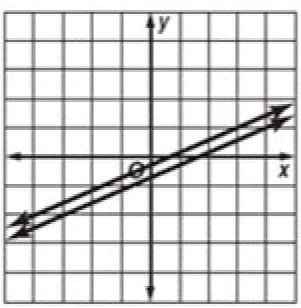
SOLUTION:
Graph both equations on the coordinate plane.



The solution of the system is (–3, 4).

14. $6x - 14y = 5$
$3x - 7y = 5$

SOLUTION:
Graph both equations on the coordinate plane.



Since the graphs are parallel, the lines never intersect. So, the system has no solution.

15. **LAWN CARE** André and Paul each mow lawns. André charges a $30 service fee and $10 per hour. Paul charges a $10 service fee and $15 per hour. After how many hours will André and Paul charge the same amount?

SOLUTION:
Let x = number of hours mowed.

$$\begin{aligned} 30 + 10x &= 10 + 15x \\ 10x - 15x &= 10 - 30 \\ -5x &= -20 \\ \frac{-5x}{-5} &= \frac{-20}{-5} \\ x &= 4 \end{aligned}$$

Therefore, André and Paul will charge the same amount for 4 hours.

Solve each system of equations by using either substitution or elimination.

16. $x + y = 6$
$3x - 2y = -2$

SOLUTION:
Substitute $y = 6 - x$ in the equation $3x - 2y = -2$.

$$\begin{aligned} 3x - 2(6 - x) &= -2 \\ 3x - 12 + 2x &= -2 \\ 5x - 12 &= -2 \\ 5x &= -2 + 12 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Substitute $x = 2$ in $y = 6 - x$.

$$\begin{aligned} y &= 6 - 2 \\ y &= 4 \end{aligned}$$

The solution is (2, 4).

17. $\begin{aligned} 5x - 2y &= 4 \\ -2y + x &= 12 \end{aligned}$

SOLUTION:

Substitute $-2y = -5x + 4$ in the equation $-2y + x = 12$.

$$\begin{aligned} -2y + x &= 12 \\ -5x + 4 + x &= 12 \\ -4x + 4 &= 12 \\ -4x &= 12 - 4 \\ -4x &= 8 \\ x &= -2 \end{aligned}$$

Substitute $x = -2$ in the equation $5x - 2y = 4$.

$$\begin{aligned} 5x - 2y &= 4 \\ 5(-2) - 2y &= 4 \\ -10 - 2y &= 4 \\ -2y &= 14 \\ y &= -7 \end{aligned}$$

The solution is (–2, –7).

18. $\begin{aligned} x + y &= 3.5 \\ x - y &= 7 \end{aligned}$

SOLUTION:

Substitute $y = x - 7$ in the equation $x + y = 3.5$.

$$\begin{aligned} x + x - 7 &= 3.5 \\ 2x - 7 &= 3.5 \\ 2x &= 10.5 \\ x &= 5.25 \end{aligned}$$

Substitute $x = 5.25$ in the equation $x - y = 7$.

$$\begin{aligned} 5.25 - y &= 7 \\ -y &= 7 - 5.25 \\ -y &= 1.75 \\ y &= -1.75 \end{aligned}$$

The solution is (5.25, –1.75).

19. $\begin{aligned} 3y - 5x &= 0 \\ 2y - 4x &= -2 \end{aligned}$

SOLUTION:

$$\begin{aligned} 3y - 5x &= 0 \\ 3y &= 5x \\ y &= \frac{5}{3}x \end{aligned}$$

Substitute $y = \frac{5}{3}x$ in the equation $2y - 4x = -2$.

$$\begin{aligned} 2\left(\frac{5}{3}x\right) - 4x &= -2 \\ \frac{10}{3}x - 4x &= -2 \\ \frac{10x - 12x}{3} &= -2 \\ -\frac{2}{3}x &= -2 \\ x &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

Substitute $x = 3$ in the equation $3y - 5x = 0$.

$$\begin{aligned} 3y - 5x &= 0 \\ 3y - 5(3) &= 0 \\ 3y - 15 &= 0 \\ 3y &= 15 \\ y &= 5 \end{aligned}$$

The solution is (3, 5).

20. **SCHOOL SUPPLIES** At an office supply store, Emilio bought 3 notebooks and 5 pens for \$13.75. If a notebook costs \$1.25 more than a pen, how much does a notebook cost? How much does a pen cost?

SOLUTION:

Let x = cost of a note book. Let y = cost of a pen. The system of equations representing the situation is:

$$\begin{aligned} 3x + 5y &= 13.75 \\ x &= y + 1.25 \end{aligned}$$

Substitute $x = y + 1.25$ in the equation $3x + 5y = 13.75$.

$$\begin{aligned} 3(y + 1.25) + 5y &= 13.75 \\ 3y + 3.75 + 5y &= 13.75 \\ 8y + 3.75 &= 13.75 \\ 8y &= 10 \\ y &= \frac{10}{8} \\ y &= 1.25 \end{aligned}$$

Substitute y = 1.25 in the equation $x = y + 1.25$.

$$\begin{aligned} x &= 1.25 + 1.25 \\ x &= 2.50 \end{aligned}$$

Therefore, the cost of a note book is \$2.50 and the cost of a pen is \$1.25.

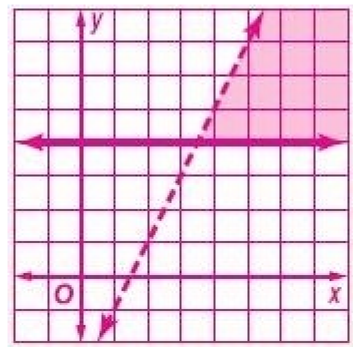
Solve each system of inequalities by graphing.

21. $y < 2x - 3$

$y \geq 4$

SOLUTION:

Graph the system of inequalities in the same coordinate plane.

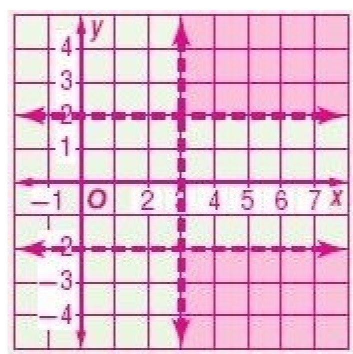


22. $|y| > 2$

$x > 3$

SOLUTION:

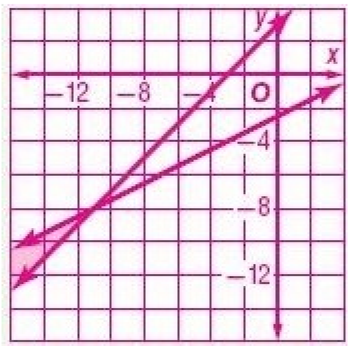
Graph the system of inequalities in the same coordinate plane.



23. $y \geq x + 3$
$2y \leq x - 5$

SOLUTION:

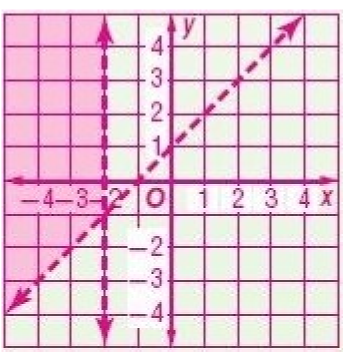
Graph the system of inequalities in the same coordinate plane.



24. $y > x + 1$
$x < -2$

SOLUTION:

Graph the system of inequalities in the same coordinate plane.



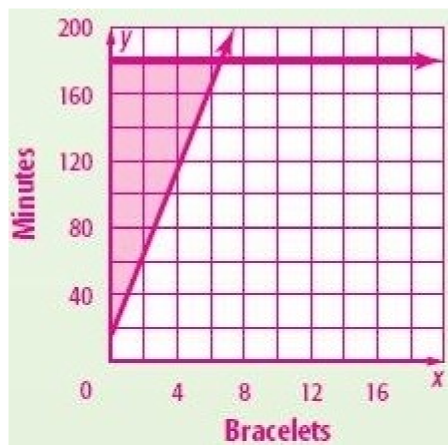
25. **JEWELRY** Payton makes jewelry to sell at her mother's clothing store. She spends no more than 3 hours making jewelry on Saturdays. It takes her 15 minutes to set up her supplies and 25 minutes to make each bracelet. Draw a graph that represents this.

SOLUTION:

Let x = number of bracelets, and y = number of minutes.
The system of inequalities representing the situation is:

$y \geq 25x + 15$
$y \leq 180$

Graph the inequalities in the same coordinate plane.



26. **FLOWERS** A florist can make a grand arrangement in 18 minutes or a simple arrangement in 10 minutes. The florist makes at least twice as many of the simple arrangements as the grand arrangements. The florist can work only 40 hours per week. The profit on the simple arrangements is \$10 and the profit on the grand arrangements is \$25. Find the number and type of arrangements that the florist should produce to maximize profit.

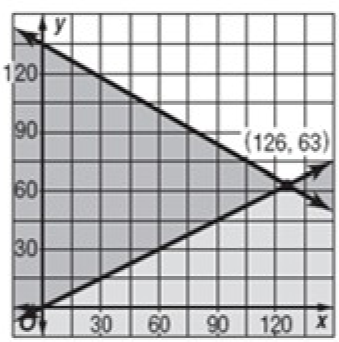
SOLUTION:

Let x be the number of simple arrangement and y be the number of grand arrangements.

$$x \geq 2y$$
$$10x + 18y \leq 2400$$

The optimize function is $f(x, y) = 10x + 25y$.

Graph the inequalities in the same coordinate plane.



The vertices of the feasible region are (0, 0), (240, 0) and (126, 63).

(x, y)	$f(x, y)$
$(0,0)$	0
$(240,0)$	$10(240)+25(0)=2400$
$(126,63)$	$10(126)+25(63)=2835$

So, to maximize the profit, the florist should produce 126 simple arrangements and 63 grand arrangements.

27. **MANUFACTURING** A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one and 1 hour in step two, and produces a profit of \$20. Each pair of indoor shoes requires 1 hour in step one and 3 hours in step 2, and produces a profit of \$15. The company has 40 hours of labor available per day for step one and 60 hours available for step two. What is the manufacturer's maximum profit? What is the combination of shoes for this profit?

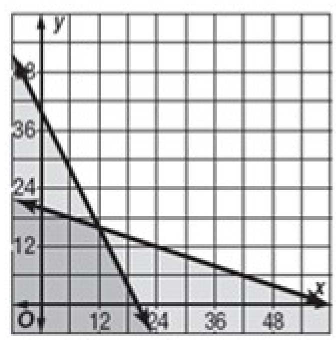
SOLUTION:

Let x be the number of pair of outdoor shoes and y be the number of pair of indoor shoes.

$$2x + y \leq 40$$
$$x + 3y \leq 60$$

The optimize function is $f(x, y) = 20x + 15y$.

Graph the inequalities in the same coordinate plane.



The vertices of the feasible region are (0, 20), (20, 0) and
(12, 16).

(x, y)	$f(x, y)$
$(0,20)$	$20(0)+15(20)=300$
$(20,0)$	$20(20)+15(0)=400$
$(12,16)$	$20(12)+15(16)=480$

The manufacturer's maximum profit is \$480. For this profit, the manufacturer has to produce 12 outdoor and 16 indoor pair of shoes.

Solve each system of equations.

28. $a - 4b + c = 3$
$b - 3c = 10$
$3b - 8c = 24$

SOLUTION:

$$\begin{aligned} b - 3c &= 10 \\ b &= 3c + 10 \end{aligned}$$

Substitute ($3c$ + 10) for b in the equation $3b - 8c = 24$.

$$\begin{aligned} 3(3c+10) - 8c &= 24 \\ 9c + 30 - 8c &= 24 \\ c + 30 &= 24 \\ c &= 24 - 30 \\ c &= -6 \end{aligned}$$

Substitute c = –6 in the equation $b = 3c + 10$.

$$\begin{aligned} b &= 3(-6) + 10 \\ &= -18 + 10 \\ &= -8 \end{aligned}$$

Substitute b = –8 and c = –6 in the equation $a - 4b + c = 3$.

$$\begin{aligned} a - 4(-8) + (-6) &= 3 \\ a + 32 - 6 &= 3 \\ a + 26 &= 3 \\ a &= 3 - 26 \\ a &= -23 \end{aligned}$$

The solution of the system is (–23, –8, –6).

29. $2x - z = 14$
$3x - y + 5z = 0$
$4x + 2y + 3z = -2$

SOLUTION:

$$\begin{aligned} 2x - z &= 14 \\ z &= 2x - 14 \end{aligned}$$

Substitute ($2x$ – 14) for z in the equations $3x - y + 5z = 0$ and $4x + 2y + 3z = -2$.

$$\begin{aligned} 3x - y + 5(2x - 14) &= 0 & 4x + 2y + 3(2x - 14) &= -2 \\ 3x - y + 10x - 70 &= 0 & 4x + 2y + 6x - 42 &= -2 \\ 13x - y &= 70 & 10x + 2y &= 40 \end{aligned}$$

$$\begin{aligned} 13x - y &= 70 \\ y &= 13x - 70 \end{aligned}$$

Substitute $y = 13x - 70$ in the equation $10x + 2y = 40$.

$$\begin{aligned} 10x + 2y &= 50 \\ 10x + 2(13x - 70) &= 50 \\ 10x + 26x - 140 &= 50 \\ 36x &= 40 + 140 \\ 36x &= 180 \\ x &= 5 \end{aligned}$$

Substitute x = 5 in the equation $y = 13x - 70$.

$$\begin{aligned} y &= 13(5) - 70 \\ &= 65 - 70 \\ &= -5 \end{aligned}$$

Substitute x = 5 in the equation $2x - z = 14$.

$$\begin{aligned} 2(5) - z &= 14 \\ 10 - z &= 14 \\ z &= 10 - 14 \\ z &= -4 \end{aligned}$$

The solution of the system is (5, –5, –4).

30. **AMUSEMENT PARKS** Dustin, Luis, and Marci went to an amusement park. They purchased snacks from the same vendor. Their snacks and how much

they paid are listed in the table. How much did each snack cost?

Name	Hot Dogs	Popcorn	Soda	Price
Dustin	1	2	3	$15.25
Luis	2	0	3	$14.00
Marci	1	2	1	$10.25

SOLUTION:

Let x = cost of a hot dog.
Let y = cost of a popcorn.
Let z = cost of a soda.

The system of equations representing the situations is:

$$\begin{aligned} x+2y+3z&=15.25 \quad \to (1)\\ 2x+3z&=14 \quad \to (2)\\ x+2y+z&=10.25 \quad \to (3)\end{aligned}$$

Use equation (3) in (1).
Substitute $x + 2y = 10.25 - z$ in equation (1).

$$\begin{aligned} 10.25-z+3z&=15.25\\ 10.25+2z&=15.25\\ 2z&=5\\ z&=2.50\end{aligned}$$

Substitute $z = 2.50$ in equation (2).

$$\begin{aligned} 2x+3z&=14\\ 2x+3(2.50)&=14\\ 2x+7.50&=14\\ 2x&=14-7.50\\ 2x&=6.50\\ x&=3.25\end{aligned}$$

Substitute $x = 3.25$ and $z = 2.50$ in the equation (1).

$$\begin{aligned} x+2y+3z&=15.25\\ 3.25+2y+3(2.50)&=15.25\\ 3.25+2y+7.50&=15.25\\ 2y+10.75&=15.25\\ 2y&=15.25-10.75\\ 2y&=4.50\\ y&=2.25\end{aligned}$$

Therefore, the cost of a hot dog is $3.25. The cost of a popcorn is $2.25 and the cost of a soda is $2.50.

Perform the indicated operations. If the matrix does not exist, write *impossible*.

31. $3\left(\begin{bmatrix}-2 & 0\\ 6 & 8\end{bmatrix}+\begin{bmatrix}1 & 9\\ -3 & -4\end{bmatrix}\right)$

SOLUTION:

$$\begin{aligned} 3\left(\begin{bmatrix}-2 & 0\\ 6 & 8\end{bmatrix}+\begin{bmatrix}1 & 9\\ -3 & -4\end{bmatrix}\right)&=3\begin{bmatrix}-1 & 9\\ 3 & 4\end{bmatrix}\\ &=\begin{bmatrix}-3 & 27\\ 9 & 12\end{bmatrix}\end{aligned}$$

32. $\begin{bmatrix}2\\ -6\end{bmatrix}-\begin{bmatrix}-3\\ 2\end{bmatrix}+\begin{bmatrix}6\\ 0\end{bmatrix}$

SOLUTION:

$$\begin{aligned} \begin{bmatrix}2\\ -6\end{bmatrix}-\begin{bmatrix}-3\\ 2\end{bmatrix}+\begin{bmatrix}6\\ 0\end{bmatrix}&=\begin{bmatrix}5\\ -8\end{bmatrix}+\begin{bmatrix}6\\ 0\end{bmatrix}\\ &=\begin{bmatrix}11\\ -8\end{bmatrix}\end{aligned}$$

33. **RETAIL** Current Fashions buys shirts, jeans and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

Item	Purchase Price	Selling Price
shirts	$15	$35
jeans	$25	$55
shoes	$30	$85

a. Write a matrix for the purchase price.
b. Write a matrix for the selling price.
c. Use matrix operations to find the profit on 1 shirt, 1 pair of jeans, and 1 pair of shoes.

SOLUTION:

a. Purchase price:

$$\begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix}$$

b. Selling price:

$$\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix}$$

c. Subtract the matrices.

$$\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix} - \begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 55 \end{bmatrix}$$

Find each product, if possible.

34. $\begin{bmatrix} 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

SOLUTION:

$$\begin{bmatrix} 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -5 \end{bmatrix} = \begin{bmatrix} 62 \end{bmatrix}$$

35. $\begin{bmatrix} -3 & 0 & 2 \\ 6 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -1 \\ -4 & 3 \\ 6 & 7 \end{bmatrix}$

SOLUTION:

$$\begin{bmatrix} -3 & 0 & 2 \\ 6 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -1 \\ -4 & 3 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} -12 & 17 \\ 82 & 26 \end{bmatrix}$$

36. $\begin{bmatrix} 2 & 11 \\ 0 & -3 \\ -6 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & -5 \\ 12 & 0 & 9 \\ 4 & -6 & 7 \end{bmatrix}$

SOLUTION:

The inner dimensions of the matrices are not equal. So, the matrices cannot be multiplied.

37. **GROCERIES** Martin bought 1 gallon of milk, 2 apples, 4 frozen dinners, and 1 box of cereal. The following matrix shows the prices for each item respectively.
[$2.59 $0.49 $5.25 $3.99]
Use matrix multiplication to find the total amount of money Martin spent at the grocery store.

SOLUTION:

$$\begin{bmatrix} 2.59 & 0.49 & 5.25 & 3.99 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 28.56 \end{bmatrix}$$

So, Martin spent $28.56.

Evaluate each determinant.

38. $\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix}$

SOLUTION:

$$\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix} = -6 - 28$$
$$= -34$$

39. $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$

SOLUTION:

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix} = 2(12-20) - 3(0+8) - 1(0+4)$$
$$= -16 - 24 - 4$$
$$= -44$$

Use Cramer's Rule to solve each system of equations.

40. $3x - y = 0$
$5x + 2y = 22$

SOLUTION:

Let $C = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$.

$$|C| = \begin{vmatrix} 3 & -1 \\ 5 & 2 \end{vmatrix}$$
$$= 11$$

$$x = \frac{\begin{vmatrix} 0 & -1 \\ 22 & 2 \end{vmatrix}}{11}$$
$$= \frac{22}{11}$$
$$= 2$$

$$y = \frac{\begin{vmatrix} 3 & 0 \\ 5 & 22 \end{vmatrix}}{11}$$
$$= \frac{66}{11}$$
$$= 6$$

Therefore, the solution is (2, 6).

41. $5x + 2y = 4$
$3x + 4y + 2z = 6$
$7x + 3y + 4z = 29$

SOLUTION:

Let $C = \begin{bmatrix} 5 & 2 & 0 \\ 3 & 4 & 2 \\ 7 & 3 & 4 \end{bmatrix}$.

$$|C| = 5(16-6) - 2(12-14) + 0(9-28)$$
$$= 50 + 4 + 0$$
$$= 54$$

$$x = \frac{\begin{vmatrix} 4 & 2 & 0 \\ 6 & 4 & 2 \\ 29 & 3 & 4 \end{vmatrix}}{54}$$
$$= \frac{108}{54}$$
$$= 2$$

$$y = \frac{\begin{vmatrix} 5 & 4 & 0 \\ 3 & 6 & 2 \\ 7 & 29 & 4 \end{vmatrix}}{54}$$
$$= -\frac{162}{54}$$
$$= -3$$

$$z = \frac{\begin{vmatrix} 5 & 2 & 4 \\ 3 & 4 & 6 \\ 7 & 3 & 29 \end{vmatrix}}{54}$$
$$= \frac{324}{54}$$
$$= 6$$

The solution is (2, −3, 6).

42. **JEWELRY** Alana paid \$98.25 for 3 necklaces and 2 pairs of earrings. Petra paid \$133.50 for 2 necklaces and 4 pairs of earrings. Use Cramer's Rule to find out how much 1 necklace costs and how much 1 pair of earrings costs.

SOLUTION:

Let x be the number of necklaces and y be the number of pairs of earrings.

$$3x + 2y = 98.25$$
$$2x + 4y = 133.50$$

Let $C = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$.

$$\begin{aligned} |C| &= \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

$$\begin{aligned} x &= \frac{\begin{vmatrix} 98.25 & 2 \\ 133.50 & 4 \end{vmatrix}}{8} \\ &= \frac{126}{8} \\ &= 15.75 \end{aligned}$$

$$\begin{aligned} y &= \frac{\begin{vmatrix} 3 & 98.25 \\ 2 & 133.50 \end{vmatrix}}{8} \\ &= \frac{204}{8} \\ &= 25.50 \end{aligned}$$

So, the cost of 1 necklace is \$15.75 and a pair of earrings is \$25.50.

Find the inverse of each matrix, if it exists.

43. $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$

SOLUTION:

$$\begin{aligned} \begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix} &= 7(2) - 3(4) \\ &= 14 - 12 \\ &= 2 \end{aligned}$$

Since the determinant is non-zero, the inverse exists.

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ -3 & 7 \end{bmatrix}$$

44. $\begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix}$

SOLUTION:

$$\begin{aligned} \begin{vmatrix} 2 & 5 \\ -5 & -13 \end{vmatrix} &= -26 + 25 \\ &= -1 \end{aligned}$$

Since the determinant is non-zero, the inverse exists.

$$\begin{aligned} A^{-1} &= \frac{1}{-1}\begin{bmatrix} -13 & -5 \\ 5 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 13 & 5 \\ -5 & -2 \end{bmatrix} \end{aligned}$$

45. $\begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$

SOLUTION:

$$\begin{aligned} \begin{vmatrix} 6 & -3 \\ -8 & 4 \end{vmatrix} &= 24 - 24 \\ &= 0 \end{aligned}$$

Since the determinant is 0, the inverse does not exist.

Use a matrix equation to solve each system of equations.

46. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

SOLUTION:

Let $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

$$A^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 8 \\ -12 \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 8 \\ -12 \end{bmatrix} \end{aligned}$$

The solution of the system is (8, −12).

47. $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

SOLUTION:

Let $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$.

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

$$\begin{aligned} \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} &= \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} &= \frac{1}{7}\begin{bmatrix} 14 \\ 7 \end{bmatrix} \\ \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{aligned}$$

The solution of the system is (2, 1).

48. **HEALTH FOOD** Heath sells nuts and raisins by the pound. Sonia bought 2 pounds of nuts and 2 pounds of raisins for $23.50. Drew bought 3 pounds of nuts and 1 pound of raisins for $22.25. What is the cost of 1 pound of nuts and 1 pound of raisins?

SOLUTION:

Let x = cost of 1 pound of nuts and y = cost of 1 pound of raisins.

$$\begin{aligned} 2x + 2y &= 23.50 \\ 3x + y &= 22.25 \end{aligned}$$

The matrix equation is $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23.50 \\ 22.25 \end{bmatrix}$.

The inverse of $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$ is $-\frac{1}{4}\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$.

$$\begin{aligned} -\frac{1}{4}\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} &= -\frac{1}{4}\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 23.50 \\ 22.25 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} &= -\frac{1}{4}\begin{bmatrix} -21 \\ -26 \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 5.25 \\ 6.50 \end{bmatrix} \end{aligned}$$

The cost of 1 pound of nuts is $5.25 and 1 pound of raisins is $6.50.