Write a quadratic equation in standard form with the given root(s).

18. $-5, \frac{1}{2}$

SOLUTION:
Write the pattern.

$$(x-p)(x-q)=0$$

Replace p and q with -5 and $\frac{1}{2}$.

$$\begin{aligned}(x-(-5))\left(x-\frac{1}{2}\right)&=0\\(x+5)\left(x-\frac{1}{2}\right)&=0\end{aligned}$$

Use the FOIL method to multiply.

$$\begin{aligned}x(x)+x\left(-\frac{1}{2}\right)+5(x)+5\left(-\frac{1}{2}\right)&=0\\x^2-\frac{1}{2}x+5x-\frac{5}{2}&=0\end{aligned}$$

Multiply each side by 2.

$$\begin{aligned}2x^2-x+10x-5&=0\\2x^2+9x-5&=0\end{aligned}$$

Factor each polynomial.

22. $32xy + 40bx - 12ay - 15ab$

SOLUTION:

Factor $8x$ from the first two terms and $-3a$ from the last two terms.

$$\begin{aligned}&32xy + 40bx - 12ay - 15ab \\ &\quad = 8x(4y + 5b) - 3a(4y + 5b)\end{aligned}$$

Factor $4y + 5b$ from the two terms.

$$\begin{aligned}&8x(4y + 5b) - 3a(4y + 5b) \\ &\quad = (4y + 5b)(8x - 3a)\end{aligned}$$

Therefore,

$$\begin{aligned}&32xy + 40bx - 12ay - 15ab \\ &\quad = (4y + 5b)(8x - 3a)\end{aligned}$$

30. $4x^2 + 29x + 30$

SOLUTION:

Here, $a = 4$, $b = 29$ and $c = 30$.

$ac = 4(30) = 120$

Find two factors of 120 whose sum is 29.

5(24) = 120 and 5 + 24 = 29

Write $29x$ as $5x + 24x$.

$$4x^2 + 29x + 30 = 4x^2 + 5x + 24x + 30$$

Factor x from the first two terms and 6 from the last two terms.

$$4x^2 + 5x + 24x + 30 = x(4x+5) + 6(4x+5)$$

Factor $4x + 5$ from the two terms.

$$x(4x+5) + 6(4x+5) = (4x+5)(x+6)$$

Therefore,

$$4x^2 + 29x + 30 = (4x+5)(x+6).$$

34. $18x^2y^2 - 24xy^2 + 36y^2$

SOLUTION:

The GCF of the three terms is $6y^2$. Factor the GCF.

$$\begin{aligned} &18x^2y^2 - 24xy^2 + 36y^2 \\ &= 6y^2(3x^2) - 6y^2(4x) + 6y^2(6) \\ &= 6y^2(3x^2 - 4x + 6) \end{aligned}$$

36. $12x^2 + 13x - 14$

SOLUTION:
Here, $a = 12$, $b = 13$ and $c = -14$.

$ac = 12(-14) = -168$

Find two factors of –168 whose sum is 13.

$-8(21) = -168$ and $-8 + 21 = 13$

Write $13x$ as $-8x + 21x$.

$$12x^2 + 13x - 14 = 12x^2 - 8x + 21x - 14$$

Factor $4x$ from the first two terms and 7 from the last two terms.

$$\begin{aligned} &12x^2 - 8x + 21x - 14 \\ &\quad = 4x(3x - 2) + 7(3x - 2) \end{aligned}$$

Factor $3x - 2$ from the two terms.

$$4x(3x-2) + 7(3x-2) = (3x-2)(4x+7)$$

Therefore,

$$12x^2 + 13x - 14 = (3x-2)(4x+7).$$

Solve each equation by factoring.

38. $x^2 + 4x - 45 = 0$

SOLUTION:

Find the factors of –45 whose sum is 4.
9(–5) = –45 and –5 + 9 = 4

Write 4*x* as –5*x* + 9*x*.

$x^2 + 4x - 45 = x^2 - 5x + 9x - 45 = 0$

Factor *x* from the first two terms and 9 from the last two terms.

$$x^2 + 5x - 9x - 45 = 0$$
$$x(x+5) - 9(x+5) = 0$$

Factor *x* + 5 from the two terms.

$(x+5)(x-9) = 0$

Use the Zero Product Property.

$$(x+5)(x-9) = 0 \Rightarrow x + 5 = 0 \quad \text{or} \quad x - 9 = 0$$
$$\Rightarrow x = -5 \quad \text{or} \quad x = 9$$

Therefore,
the roots are –5 and 9.

40. $x^2 = 121$

SOLUTION:

Write the equation with right side equal to zero.

$x^2 - 121 = 0$

Use the identity $a^2 - b^2 = (a + b)(a - b)$ to factor $x^2 - 121$.

$$x^2 - 121 = 0$$
$$(x+11)(x-11) = 0$$

Use the Zero Product Property.

$$(x+11)(x-11) = 0$$
$$\Rightarrow x + 11 = 0 \quad \text{or} \quad x - 11 = 0$$
$$\Rightarrow x = -11 \quad \text{or} \quad x = 11$$

Therefore, the roots are –11 and 11.

42. $-3x^2 - 10x + 8 = 0$

SOLUTION:
Factor out –1.

$$\begin{aligned} -1\left(3x^2 + 10x - 8\right) &= 0 \\ 3x^2 + 10x - 8 &= 0 \end{aligned}$$

Now factor $3x^2 + 10x - 8$.

Here, $a = 3$, $b = 10$ and $c = -8$.

$ac = 3(-8) = -24$

Find two factors of –24 whose sum is 10.

$12(-2) = -24$ and $12 + (-2) = 10$

Write $10x$ as $12x + (-2x)$.

$$3x^2 + 10x - 8 = 3x^2 + 12x - 2x - 8$$

Factor $3x$ from the first two terms and –2 from the last two terms.

$$\begin{aligned} 3x^2 + 12x - 2x - 8 &= 0 \\ 3x(x+4) - 2(x+4) &= 0 \end{aligned}$$

Factor $x + 4$ from the two terms.

$$\begin{aligned} 3x(x+4) - 2(x+4) &= 0 \\ (x+4)(3x-2) &= 0 \end{aligned}$$

Use the Zero Product Property.

$$\begin{aligned} &(x+4)(3x-2) = 0 \\ &\Rightarrow x + 4 = 0 \quad \text{or} \quad 3x - 2 = 0 \\ &\qquad \Rightarrow x = -4 \quad \text{or} \quad x = \frac{2}{3} \end{aligned}$$

Therefore, the roots are -4 and $\frac{2}{3}$.

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

SOLUTION:

Let x be the length of the one of the legs. Then the length of the hypotenuse is $3x + 4$ and that of the other leg is $3x + 3$.

By the Pythagorean Theorem, the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

$$(3x+4)^2 = (3x+3)^2 + x^2$$

Simplify and write in the standard form of a quadratic equation.

$$\begin{aligned}(3x)^2 + 2(3x)(4) + (4)^2 &= (3x)^2 + 2(3x)(3) + (3)^2 + (x)^2 \\ 9x^2 + 24x + 16 &= 9x^2 + 18x + 9 + x^2 \\ x^2 - 6x - 7 &= 0\end{aligned}$$

Find two factors of –7 whose sum is –6.

1(–7) = –7 and 1 + (–7) = –6

Write $-6x$ as $x + (-7x)$.

$$\begin{aligned}x^2 - 6x - 7 &= 0 \\ x^2 + x - 7x - 7 &= 0\end{aligned}$$

Factor x from the first two terms and –7 from the last two terms.

$$x(x+1) - 7(x+1) = 0$$

Factor $x + 1$ from the two terms.

$$(x+1)(x-7) = 0$$

Use the Zero Product Property.

$$\begin{aligned}(x+1)(x-7) = 0 &\Rightarrow x+1 = 0 \quad \text{or} \quad x-7 = 0 \\ &\Rightarrow x = -1 \quad \text{or} \quad x = 7\end{aligned}$$

But x is a length; it cannot be negative. So, $x = 7$.

Therefore, the lengths of the sides are 7 cm, 24 cm, and 25 cm.

45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.

SOLUTION:
Let the numbers be $2n$ and $2(n + 1)$.

Their product is 624.

$$2n\left(2\left(n+1\right)\right)=624$$
$$4n^2+4n-624=0$$

Here, $a = 4$, $b = 4$ and $c = 624$.

$ac = 4(624) = 2496$

Find two factors of 2496 whose sum is 4.
52(–48) = 2496 and 52 + (–48) = 4

Write $4n$ as $52n - 48n$.

$$4n^2+4n-624=0$$
$$4n^2+52n-48n-624=0$$

Factor $4n$ from the first two terms and –48 from the last two terms.

$$4n^2+52n-48n-624=0$$
$$4n\left(n+13\right)-48\left(n+13\right)=0$$

Factor $n + 13$ from the two terms.

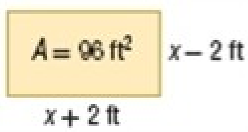
$$\left(4n-48\right)\left(n+13\right)=0$$

Use the Zero Product Property.

$$\left(4n-48\right)\left(n+13\right)=0 \Rightarrow 4n-48=0 \quad \text{or} \quad n+13=0$$
$$\Rightarrow n=12 \qquad \text{or} \quad n=-13$$

When $n = 12$, the numbers are 24 and 26.
When $n = -13$, the numbers are –24 and –26.

GEOMETRY Find *x* and the dimensions of each rectangle.

46. 

SOLUTION:

The area of a rectangle of length *l* and width *w* is $l \times w$.

Here, $l = x + 2$, $w = x - 2$, and area = 96.

$$\begin{aligned}(x+2)(x-2) &= 96\\ x^2 - 4 &= 96\\ x^2 &= 100\\ x &= \pm 10\end{aligned}$$

When $x = -10$, the dimensions of the rectangle becomes negative. So, $x = 10$.

The length of the rectangle is 12 ft and width is 8 ft.

Solve each equation by factoring.

60. $27x^2 + 5 = 48x$

SOLUTION:
Write the equation with right side equal to zero.

$27x^2 - 48x + 5 = 0$

Find factors of 27(5) = 135 whose sum is –48.

–45(–3) = 135 and –45 + (–3) = –48

$$
\begin{aligned}
27x^2 - 45x - 3x + 5 &= 0 \\
9x(3x-5) - 1(3x-5) &= 0 \\
(3x-5)(9x-1) &= 0 \\
\Rightarrow 3x - 5 = 0 \text{ or } 9x - 1 &= 0 \\
\Rightarrow x = \frac{5}{3} \quad or \;\; x &= \frac{1}{9}
\end{aligned}
$$

Therefore, the roots are $\frac{5}{3}$ and $\frac{1}{9}$.