UCHS AP Calculus 2.3 HW Solutions

1. $\lim_{x\to 9} x$

SOLUTION $\lim_{x\to 9} x = 9.$

7. $\lim_{x\to 0.2} (3x+4)$

SOLUTION Using the Sum Law and the Constant Multiple Law:

$$\lim_{x\to 0.2}(3x+4) = \lim_{x\to 0.2} 3x + \lim_{x\to 0.2} 4$$
$$= 3\lim_{x\to 0.2} x + \lim_{x\to 0.2} 4 = 3(0.2)+4 = 4.6.$$

9. $\lim_{x\to -1}(3x^4 - 2x^3 + 4x)$

SOLUTION Using the Sum Law, the Constant Multiple Law and the Powers Law:

$$\lim_{x\to -1}(3x^4 - 2x^3 + 4x) = \lim_{x\to -1} 3x^4 - \lim_{x\to -1} 2x^3 + \lim_{x\to -1} 4x$$
$$= 3\lim_{x\to -1} x^4 - 2\lim_{x\to -1} x^3 + 4\lim_{x\to -1} x$$
$$= 3(-1)^4 - 2(-1)^3 + 4(-1) = 3+2-4 = 1.$$

11. $\lim_{x\to 2}(x+1)(3x^2-9)$

SOLUTION Using the Product Law, the Sum Law and the Constant Multiple Law:

$$\lim_{x\to 2}(x+1)\left(3x^2-9\right) = \left(\lim_{x\to 2} x + \lim_{x\to 2} 1\right)\left(\lim_{x\to 2} 3x^2 - \lim_{x\to 2} 9\right)$$
$$= (2+1)\left(3\lim_{x\to 2} x^2 - 9\right)$$
$$= 3(3(2)^2-9) = 9.$$

13. $\lim_{t\to 4}\frac{3t-14}{t+1}$

SOLUTION Using the Quotient Law, the Sum Law and the Constant Multiple Law:

$$\lim_{t\to 4}\frac{3t-14}{t+1} = \frac{\lim_{t\to 4}(3t-14)}{\lim_{t\to 4}(t+1)} = \frac{3\lim_{t\to 4} t - \lim_{t\to 4} 14}{\lim_{t\to 4} t + \lim_{t\to 4} 1} = \frac{3\cdot 4 - 14}{4+1} = -\frac{2}{5}.$$

15. $\lim_{y\to \frac{1}{4}}(16y+1)(2y^{1/2}+1)$

SOLUTION Using the Product Law, the Sum Law, the Constant Multiple Law and the Powers Law:

$$\lim_{y\to \frac{1}{4}}(16y+1)(2y^{1/2}+1) = \left(\lim_{y\to \frac{1}{4}}(16y+1)\right)\left(\lim_{y\to \frac{1}{4}}(2y^{1/2}+1)\right)$$
$$= \left(16\lim_{y\to \frac{1}{4}} y + \lim_{y\to \frac{1}{4}} 1\right)\left(2\lim_{y\to \frac{1}{4}} y^{1/2} + \lim_{y\to \frac{1}{4}} 1\right)$$
$$= \left(16\left(\frac{1}{4}\right)+1\right)\left(2\left(\frac{1}{2}\right)+1\right) = 10.$$

17. $\lim_{y\to 4} \frac{1}{\sqrt{6y+1}}$

SOLUTION Using the Quotient Law, the Powers Law, the Sum Law and the Constant Multiple Law:

$$\lim_{y\to 4} \frac{1}{\sqrt{6y+1}} = \frac{1}{\lim_{y\to 4} \sqrt{6y+1}} = \frac{1}{\sqrt{6 \lim_{y\to 4} y + 1}}$$
$$= \frac{1}{\sqrt{6(4)+1}} = \frac{1}{5}.$$

19. $\lim_{x\to -1} \frac{x}{x^3+4x}$

SOLUTION Using the Quotient Law, the Sum Law, the Powers Law and the Constant Multiple Law:

$$\lim_{x\to -1} \frac{x}{x^3+4x} = \frac{\lim_{x\to -1} x}{\lim_{x\to -1} x^3 + 4 \lim_{x\to -1} x} = \frac{-1}{(-1)^3 + 4(-1)} = \frac{1}{5}.$$

21. $\lim_{t\to 25} \frac{3\sqrt{t} - \frac{1}{5}t}{(t-20)^2}$

SOLUTION Using the Quotient Law, the Sum Law, the Constant Multiple Law and the Powers Law:

$$\lim_{t\to 25} \frac{3\sqrt{t} - \frac{1}{5}t}{(t-20)^2} = \frac{3\sqrt{\lim_{t\to 25} t} - \frac{1}{5}\lim_{t\to 25} t}{\left(\lim_{t\to 25} t - 20\right)^2} = \frac{3(5) - \frac{1}{5}(25)}{5^2} = \frac{2}{5}.$$

23. $\lim_{t\to \frac{3}{2}} (4t^2 + 8t - 5)^{3/2}$

SOLUTION Using the Powers Law, the Sum Law and the Constant Multiple Law:

$$\lim_{t\to \frac{3}{2}} (4t^2 + 8t - 5)^{3/2} = \left(4 \lim_{t\to \frac{3}{2}} t^2 + 8 \lim_{t\to \frac{3}{2}} t - 5\right)^{3/2} = (9 + 12 - 5)^{3/2} = 64.$$

25. Use the Quotient Law to prove that if $\lim_{x\to c} f(x)$ exists and is nonzero, then

$$\lim_{x\to c} \frac{1}{f(x)} = \frac{1}{\lim_{x\to c} f(x)}$$

SOLUTION Since $\lim_{x\to c} f(x)$ is nonzero, we can apply the Quotient Law:

$$\lim_{x\to c} \left(\frac{1}{f(x)}\right) = \frac{\left(\lim_{x\to c} 1\right)}{\left(\lim_{x\to c} f(x)\right)} = \frac{1}{\lim_{x\to c} f(x)}.$$

27. $\lim_{x\to -4} f(x)g(x)$

SOLUTION $\lim_{x\to -4} f(x)g(x) = \lim_{x\to -4} f(x) \lim_{x\to -4} g(x) = 3 \cdot 1 = 3.$

29. $\lim_{x\to -4} \frac{g(x)}{x^2}$

SOLUTION Since $\lim_{x\to -4} x^2 \neq 0$, we may apply the Quotient Law, then applying the Powers Law:

$$\lim_{x\to -4} \frac{g(x)}{x^2} = \frac{\lim_{x\to -4} g(x)}{\lim_{x\to -4} x^2} = \frac{1}{\left(\lim_{x\to -4} x\right)^2} = \frac{1}{16}.$$

31. Can the Quotient Law be applied to evaluate $\lim_{x\to 0} \frac{\sin x}{x}$? Explain.

SOLUTION The limit Quotient Law *cannot* be applied to evaluate $\lim_{x\to 0} \frac{\sin x}{x}$ since $\lim_{x\to 0} x = 0$. This violates a condition of the Quotient Law. Accordingly, the rule *cannot* be employed.

33. Give an example where $\lim_{x\to 0} (f(x) + g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

SOLUTION Let $f(x) = 1/x$ and $g(x) = -1/x$. Then $\lim_{x\to 0} (f(x) + g(x)) = \lim_{x\to 0} 0 = 0$ However, $\lim_{x\to 0} f(x) = \lim_{x\to 0} 1/x$ and $\lim_{x\to 0} g(x) = \lim_{x\to 0} -1/x$ do not exist.